

# ENDGAME PERFORMANCE TRADEOFF STUDY OF A SPECIAL CLASS OF INTERCEPTORS

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## INTRODUCTION

This study is concerned with investigating the endgame performance of a specific class of ballistic missile defense (BMD) interceptors: small, agile missiles configured to destroy re-entry vehicles (RVs) by direct impact deep in the atmosphere. The endgame performance evaluation is carried out by analyzing the sensitivity of the miss distance statistics (mean and standard deviation) to various system parameters. The system parameters of interest are sensor accuracy, interceptor response time, physical dimensions (mass and shape), sensor aperture, and sensor carrier frequency. One fixed parameter is the interceptor guidance law; the one used is of the predictive proportional navigation type.

The analytical tools used in this effort are the Cramer-Rao lower bound technique and a nonlinear covariance analysis. The first provides a measure of the interceptor's tracking performance: a lower bound on the estimation error. Given the bound time history, the nonlinear covariance equations of the engagement are propagated from handover to intercept. This procedure results in the final miss distance statistics

In this paper we will present partial results of the interception tradeoff analysis. A more detailed discussion can be found elsewhere [1]. The following tradeoff curves will be presented:

1. sensor accuracy versus missile response time for several values of equal-miss-distance (rms);
2. interceptor response time versus mass (during homing phase) parameterized by L/D (Length-to-Diameter);
3. missile mass versus sensor aperture parameterized by L/D; and
4. sensor aperture versus sensor accuracy parametrized by carrier frequency.

These four sets of curves are then integrated into a single figure (Fig. 1) such that an overall tradeoff analysis can be performed. In the following we develop the equations leading to Fig. 1.

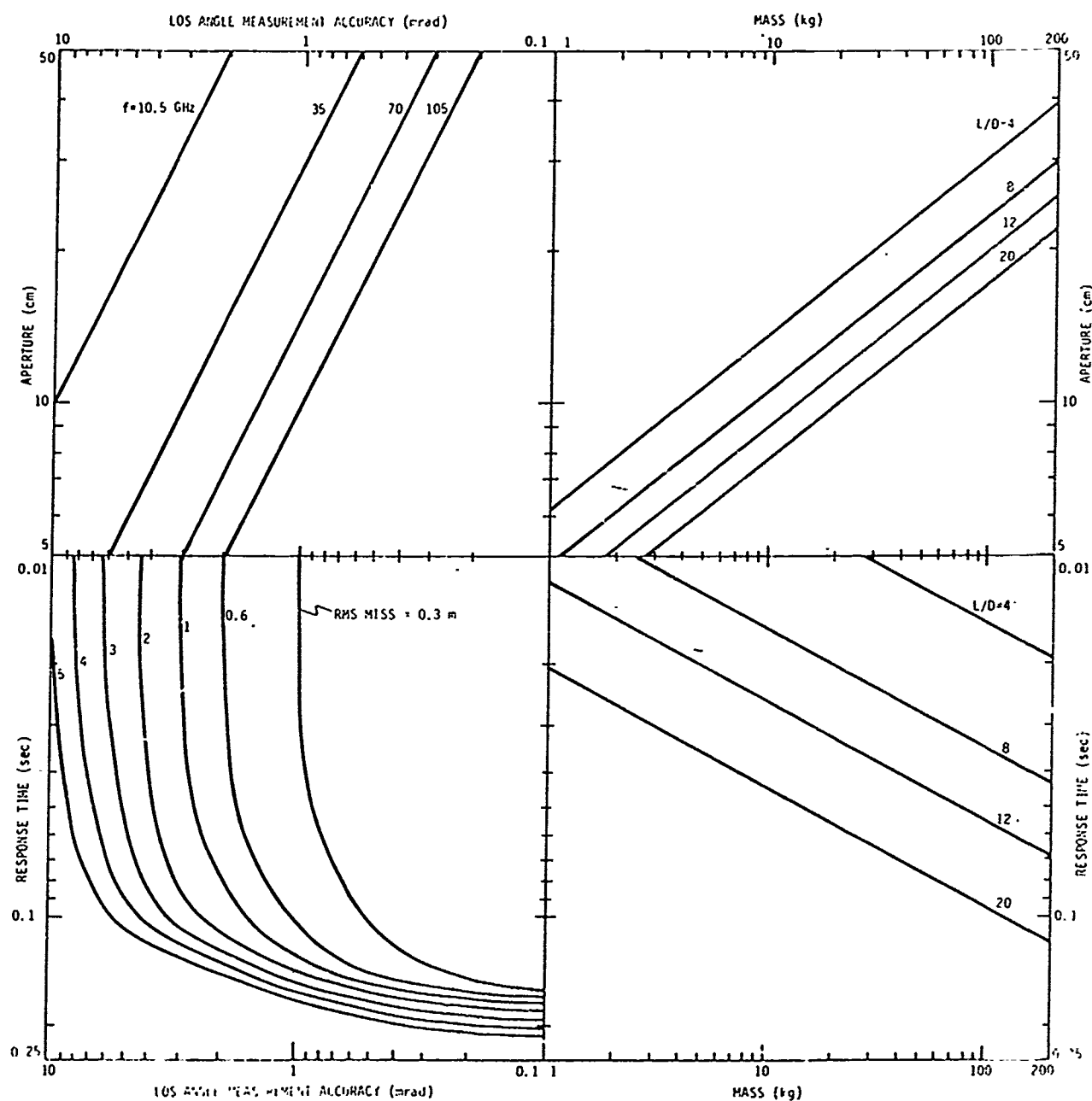


Figure 1. Overall Tradeoff Analysis Curves.

## SENSOR ACCURACY/RESPONSE TIME TRADEOFF\*

Clearly, if a specific miss-distance (rms) is to be achieved, several interceptor parameters must be traded off. We fix the handover volume, the interceptor acceleration limit, and the data rate at their nominal values [1], and trade off the LOS angle accuracy ( $\sigma_\phi$ ) versus the interceptor response time ( $\tau_m$ ) such that a particular rms miss value is attained. The equations leading to these curves are beyond the scope of this paper and can be found elsewhere. The curves are constructed in the lower-left quadrant of Fig. 1. Observe that, for a given miss distance, higher sensor accuracy permits larger time lags, and, on the other hand, higher sensor noise requires faster response times.

## RESPONSE TIME/INTERCEPTOR MASS TRADEOFF

The dominant factors affecting the response time are the missile's mass and its aerodynamic properties. In this paper we consider a cylindrical and, typically, long missile, similar to the design of Vought Corporation. Such a missile is often characterized by its Length-to-Diameter ratio, or L/D. Given an L/D value, the single dominant parameter which determines the missile's response time is the mass, M.<sup>†</sup>

The response between the commanded and the attained lateral acceleration of a typical missile is dominated by its longitudinal axis short-period natural frequency  $\omega_n$ . This frequency is given by [2].

$$\omega_n = \left[ \frac{\frac{D}{2v_m} C_{M_q} C_{Z_\alpha} - \frac{Mv_m}{Sq} C_{M_\alpha}}{\frac{I_y}{SqD} \quad \frac{Mv_m}{Sq}} \right]^{1/2} \quad (1)$$

\* Response time, as defined in this context, is equivalent to the airframe dynamic time lag  $\tau_m$ . It should be interpreted as the time it takes to attain  $a_L = (1 - e^{-1})a^c = 0.63a^c$ , where  $a^c$  is an acceleration step command.

<sup>†</sup> Usually, the response time is linked to the missile's closed-loop control bandwidth. From discussions with Vought Corporation's technical staff, however, it has been established that the control loop is used mainly for stabilization, and that its contribution to the missile's response time can be neglected.

where

$D$  = missile's diameter

$S$  = characteristic area ( $= \pi/4 D^2$ ),

$v_m$  = missile's velocity

$q$  = dynamic pressure ( $= 1/2 \rho v_m^2$ ,  $\rho$  = air density),

$M$  = missile's mass, and

$I_y$  = pitch axis moment of inertia;

and  $C_{M_q}$ ,  $C_{Z_\alpha}$ , and  $C_{M_\alpha}$  are the missile's longitudinal stability derivatives, defined as

$C_{M_q}$  = damping in pitch,

$C_{Z_\alpha}$  = slope of the normal force curve, and

$C_{M_\alpha}$  = static longitudinal stability.

From discussions with Vought technical personnel it has been determined that the typical cylindrical missile design is characterized by stability derivative values such that

$$\frac{D}{2v_m} C_{M_q} C_{Z_\alpha} \ll - \frac{Mv_m}{Sq} C_{M_\alpha} \quad (2)$$

Thus, Eq. 1 can be simplified, viz.,

$$\omega_n = \left[ \frac{-C_{M_\alpha} \cdot SqD}{I_y} \right]^{1/2} \quad (3)$$

This equation can be further simplified if we make use of the definition of  $C_{M_\alpha}$  [2]

$$C_{M_\alpha} = \frac{1}{SqD} M_\alpha \quad (4)$$

where  $M_\alpha$  is the sensitivity of the pitch axis moment to the missile angle of attack. Now,

$$\omega_n^{-1} = \left[ \frac{-M_\alpha}{I_y} \right]^{-1/2} \quad (5)$$

which, for the Vought design, is typically associated with the 90 percent value response time. Thus we may write

$$\tau_m = \frac{0.63}{0.9} \cdot \frac{1}{\omega_n} = 0.7 \left[ \frac{I_y}{-M_\alpha} \right]^{1/2} \quad (6)$$

or

$$\tau_m = 0.7 \left[ \frac{I_y}{-C_{M_\alpha} S q D} \right]^{1/2} \quad (7)$$

Note, the moment of inertia is given by

$$I_y = \frac{1}{12} M \left[ 3 \left( \frac{D}{2} \right)^2 + L^2 \right] \quad (8)$$

where L is the missile's length. Since we will consider L/D values in excess of 4 (a typical value is 12), Eq. 8 can be approximated by

$$I_y = \frac{1}{12} M L^2 \quad (9)$$

Substituting Eq. 9 and the equations for the characteristic area S, and the dynamic pressure q into Eq. 7, yields

$$\begin{aligned} \tau_m &= 0.7 \left[ \frac{\frac{1}{12} M L^2}{-C_{M_\alpha} \frac{\pi}{4} D^2 \cdot \frac{1}{2} \rho v_m^2 D} \right]^{1/2} \\ &= 0.7 \left[ \frac{2}{3\pi (-C_{M_\alpha}) \rho v_m^2} \right]^{1/2} D^{-1/2} \cdot M^{1/2} \cdot (L/D) \end{aligned} \quad (10)$$

In order to obtain  $\tau_m$  in the requisite form the missile's diameter must be expressed in terms of its mass and its L/D ratio. Clearly,

$$M = \rho_m \cdot \frac{\pi}{4} D^2 \cdot L = \rho_m \cdot \frac{\pi}{4} \left( \frac{L}{D} \right) D^3, \quad (11)$$

where  $\rho_m$  is the missile average density; thus

$$D = \left( \frac{\pi}{4} \rho_m \right)^{-1/3} \left[ \frac{M}{(L/D)} \right]^{1/3} \quad (12)$$

Substituting Eq. 12 into Eq. 10 results in

$$\tau_m = 0.7 \left[ \frac{2}{3\pi(-C_{M_\alpha})\rho v_m^2} \right]^{1/2} \left( \frac{\pi}{4} \rho_m \right)^{1/6} \cdot (L/D)^{7/6} \cdot M^{1/3} \quad (13)$$

We use the following typical values for the constants in Eq. 13:

$$\rho = 0.002 \text{ lb/ft}^3 \text{ (note, the engagement is assumed to take place at a low altitude),}$$

$$\rho_m = 0.4 \text{ gr/cm}^3 \text{ (note missile's low mass during homing phase),}^*$$

$$C_{M_\alpha} = -0.7,^* \text{ and}$$

$$v_m = 1500 \text{ m/sec (nominal value).}$$

Therefore

$$\tau_m = 6.6 \cdot 10^{-4} (L/D)^{7/6} M^{1/3} \text{ (in sec; } M \text{ in kg)} \quad (14)$$

One can plot now the desired equal L/D curves of  $\tau_m$  versus M, as shown in the lower right quadrant of Fig. 1. The L/D values considered are 4, 8, 12, and 20.

#### INTERCEPTOR MASS/SENSOR APERTURE TRADEOFF

A typical sensor aperture has a diameter (P) which is approximately two-thirds of the interceptor's diameter (D). Thus,

$$M = \rho_m \frac{\pi}{4} \left( \frac{3}{2} P \right)^2 \cdot L = \rho_m \frac{\pi}{4} \cdot \left( \frac{3}{2} P \right)^3 (L/D) \quad (15)$$

$$P = \left( \frac{32}{27\pi\rho_m} \right)^{1/3} (L/D)^{-1/3} M^{1/3} \quad (16)$$

and with  $\rho_m = 0.4 \text{ gr/cm}^3$ ,

$$P = 9.8 (L/D)^{-1/3} M^{1/3} \text{ (in cm; } M \text{ in kg)} \quad (17)$$

\*Numbers obtained from Vought Corporation.

This equation is illustrated in the upper-right quadrant of Fig. 1 for  $L/D = 4, 8, 12, \text{ and } 20$ .

#### SENSOR APERTURE/LOS ANGLE ACCURACY TRADEOFF

A widely used empirical formula for the accuracy of a LOS angle measurement is

$$\sigma_{\phi} = \frac{K}{f \cdot p}, \quad (18)$$

where  $f$  is the carrier frequency,  $P$  is the sensor aperture, and  $K$  is a constant. From experience, a 15 cm aperture in conjunction with a frequency of 35 GHz will result in about 2 mrad sensor accuracy. Thus, we may assume that

$$\begin{aligned} K &= \sigma_{\phi} \cdot f \cdot p \\ K &= 2 \cdot 10^{-3} \cdot 35 \cdot 10^9 \cdot 15 = 05 \cdot 10^9 \cdot \text{rad} \cdot \text{Hz} \cdot \text{cm} \end{aligned} \quad (19)$$

and

$$\sigma_{\phi} = \frac{1.05 \cdot 10^6}{f \cdot p} \quad (\text{in mrad; } f \text{ in Hz, and } P \text{ in cm}) \quad (20)$$

This result is shown in the upper-left quadrant of Fig. 9-1, for  $f = 10.5, 35, 70, \text{ and } 105 \text{ GHz}$ .

#### OVERALL TRADEOFF ANALYSIS

A possible application of Fig. 1 to an overall tradeoff analysis is illustrated in Fig. 2. Suppose that it is desired to achieve a 0.3 m rms miss distance, and it is assumed that the interceptor will have a 50 msec response time. This design parameter is indicated by point 1 in the lower-left quadrant, and the resultant LOS angle measurement accuracy is  $\sim 0.85 \text{ mrad}$ . Next, we choose a typical  $L/D$  value of 12, thus defining point 2 in the lower-right quadrant. By extending a straight line from point 2 to the upper-right quadrant we find that the missile's corresponding mass is  $\sim 75 \text{ kg}$ . Point 3 is then defined, in the upper-right quadrant, by the intersection of this line with the  $L/D = 12$  curve. Point 3 also corresponds to an aperture of about 18 cm. The final point (4) in the upper-left quadrant is obtained from points 1 and 3, as shown, and it determines the required carrier frequency. In the case at hand, point 4 defines a carrier frequency which is slightly lower than 70 GHz.

This example illustrates a possible application of this approach. Naturally, one may start at any quadrant and proceed in a similar

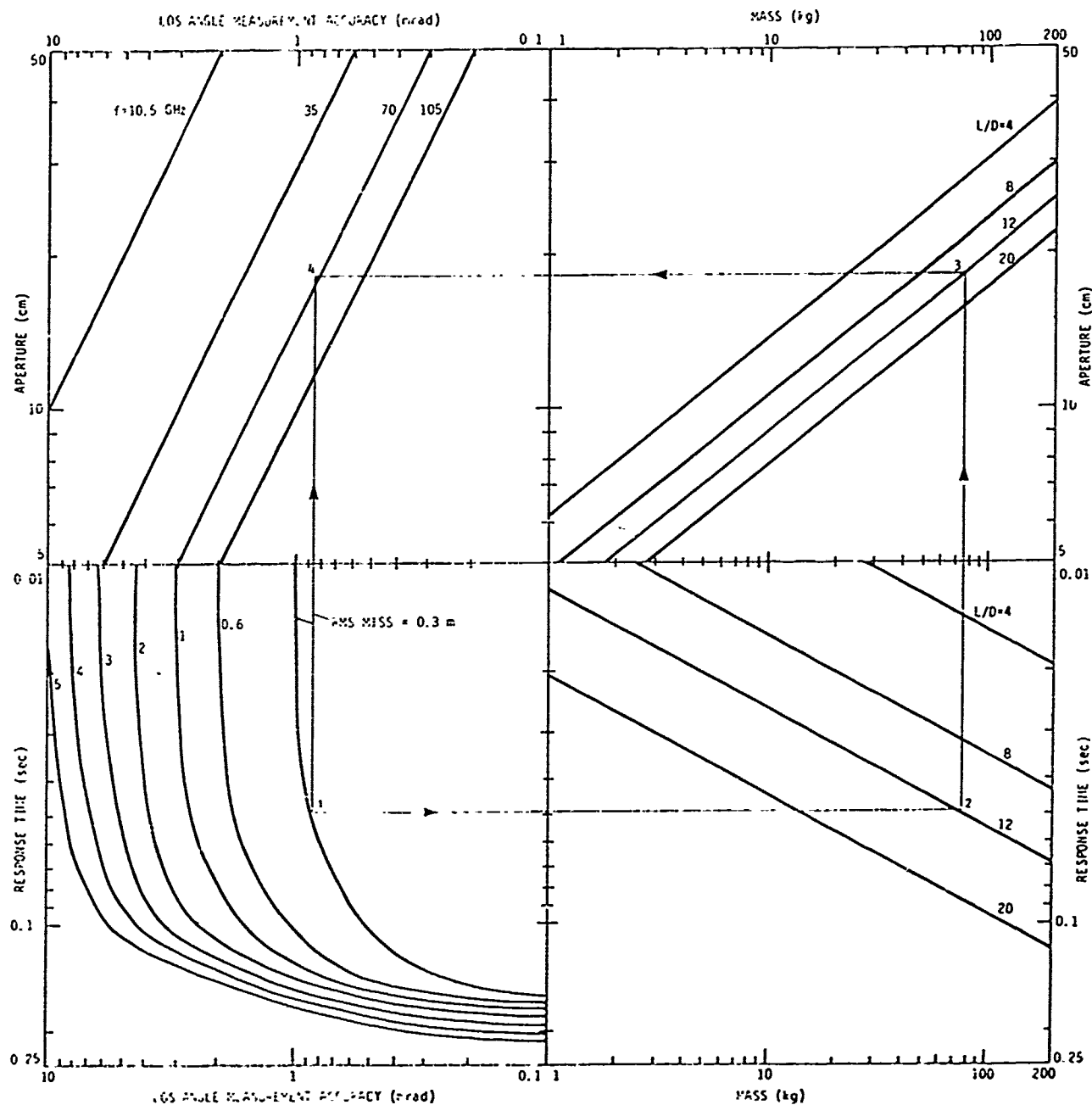


Figure 2. Tradeoff Analysis - A Representative Example.



manner through the remaining three. Note, however, that neither the lower-right nor the upper-right quadrants should be the last ones in such analysis, as both must assume the same  $L/D$ .

## CONCLUSIONS

In reference [1], a comprehensive statistical technique for analysing the interception performance of a ballistic missile in an NNK engagement was developed. The method employs analytical tools such as Cramer-Rao lower bound and nonlinear covariance analysis of the engagement kinematics. Assuming a specific engagement geometry and guidance law, a sensitivity analysis of the miss distance statistics to various system parameters was performed.

In this paper, the endgame performance analysis has been extended to a more general tradeoff study. This extension addressed the tradeoffs associated with interceptor parameters such as mass, response time, physical dimensions ( $L/D$ ), active sensor aperture, carrier frequency, and measurement accuracy. The tradeoff curves included:

1. sensor accuracy versus missile response time for several values of equal-miss-distance (rms),
2. interceptor response time versus mass (during homing phase) parametrized by  $L/D$ ,
3. missile mass versus sensor aperture parametrized by  $L/D$ , and
4. sensor aperture versus sensor accuracy parametrized by carrier frequency.

## REFERENCES

1. Korn, J., "Endgame Performance Tradeoff Analysis for NNK Interceptors", TR-132, ALPHATECH, Inc., 3 New England Executive Park, Burlington, Massachusetts 01803.
2. Blakelock, J.H., Automatic Control of Aircraft and Missiles, John Wiley and Sons, Inc., 1965.

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